10. NOISE

Naturally occurring and man-made noises have to be considered generally as the electromagnetic environment of telecommunication systems. The solutions of different telecommunication problems is made more difficult by the fact that noises are superposed to the useful signal during signal processing. In the following chapters, basic noise terms and relations will be discussed. We will concentrate almost exclusively on the thermal noise, for other types of noises we are confined to the conceptual introduction of the phenomena.

10.1. Thermal noise

Noises can be classified by their stochastic behaviour (by amplitude distribution and frequency dependence), by their physical nature, e.t.c. There is an important class of noises originating from the incidental fluctuation around a thermal balance. The properties of thermal noise have been exactly derived by physicists, using quantum mechanics approach. We will simply quote the appropriate results in the following discussion. Output noise power of an arbitrary passive network with absolute temperature *T* is characterized by

$$S(f) = \frac{hf}{e^{\frac{hf}{kT}} - 1} \tag{10.1}$$

where $S(f)$ is the power spectral density, f is the frequency, h is the Plank's constant and k is the Boltzman constant. Equation (10.1.) is called Plank's law and it determines the maximum available output noise power of a physical system as the function of frequency.

At low frequencies, the exponent in the denominator is so small that the exponential function can be approximated by the first two elements of its Taylor-series

$$S(f) = \frac{hf}{e^{\frac{hf}{kT}} - 1} = \frac{hf}{\left(1 + \frac{hf}{kT} + \ldots\right) - 1} \cong k\,T \tag{10.2}$$

The lower the frequency and the higher the temperature are, the more precise is the approximation above. E.g., if the frequency is 30 GHz and the temperature 30°K, the error caused by the approximation is less than 0.1 dB. In the optical region, conditions are quite different, e.g. at 200 THz frequency and 2000°K temperature the noise is only one thousandth part of that computed from the approximation. That is as far as frequency response of the noise is concerned, the ‘radio’ range can be distinguished from the ‘optical’ range. In the radio range, the power spectral density of thermal noise is practically constant (kT) while in the optical region the thermal noise can be neglected.

So far we have not discussed the precise meaning of 'temperature', supposing that it is equivalent to the physical absolute temperature. In the most simple case, it really is, but in more complex systems the temperature may be different so that an average should be defined to obtain the resulting conditions. Let us postpone the detailed description of this definition and let the so called *equivalent noise temperature* be used as the quantity which characterizes the noise source as if it were the real source temperature.

10.2. Noise Characteristics of a Transmission System

Because the transmission system adds its own noise to the processed signal, there are three components appearing at the output: the useful signal, the noise of the input source and the system's self noise . A normalization is used for system computations to obtain the instantaneous power by squaring the normalized quantity.

E.g. if voltage u is measured on a resistance R then

$$s = \frac{u}{\sqrt{R}} \tag{10.3}$$

is given as the normalized value of the signal. Suppose the frequency response is constant in the transmission range, then

$$s_o = A \cdot s_i + A \cdot n_i + n_1 \tag{10.4}$$

where s_o is the output signal, A is the transfer function of the system, s_i is the input signal, n_i is the input noise and n_1 is the noise generated by the system. The output power is then

$$P_0 = \overline{s_0^2} = \overline{\left(A \cdot s_i + A \cdot n_i + n_1\right)^2} \tag{10.5}$$

Since the components are independent, the expected value of their products is zero, i.e.

$$P_o = A^2 \cdot \overline{s_i^2} + A^2 \cdot \overline{n_i^2} + n_1^2 \tag{10.6}$$

Taking into account that the square of the transfer factor is identical with the power gain factor G, we receive

$$P_o = G \cdot P_{si} + G \cdot P_{ni} + P_n \tag{10.7}$$

Equation (10.7.) seems to be obvious as it states that the output power is the sum of three powers (that of the amplified input signal and amplified input noise, and that of the system noise). This is true only if the three sources are independent otherwise their mutual relation shall be taken into consideration as well.

Output power can also be expressed by the source noise temperature

$$P_o = G \cdot P_{si} + G \cdot B \cdot k \cdot T + P_n \tag{10.8}$$

or

$$P_o = G \cdot P_{si} + G \cdot B \cdot k \cdot \left(T + \frac{P_n}{GBk}\right) \tag{10.9}$$

As it turns out of equation (10.9), the influence of the system can be taken into consideration as if the temperature had risen

$$P_o = G \cdot P_{si} + G \cdot B \cdot k \cdot (T + T_{red}) \tag{10.10}$$

thus the second temperature is called *effective (input) noise temperature*. This is a very important quantity as it unanimously defines the system's contribution to the resulting noise.

Besides the effective noise temperature, *noise figure* is one of the most commonly used terms to characterize the noise properties of a block. Noise figure is defined as the ratio of the output noise power related to the amplified input noise power, provided the temperature of the input noise source is T_o = 290°K. The noise figure formula is

$$F = \frac{P_{n0}}{GP_{ni}}\Big|_{T_0} \qquad (10.11)$$

As the output power of the useful signal is G times the value of the input signal power, the noise figure shows also the degradation of the signal-to-noise ratio when the temperature of the input noise source is supposed to be 290°K. Noise figure is usually given in dB (as 10·lgF). It should be emphasized that 290°K as temperature reference is an important part of the definition of the noise figure, without it the noise figure would not be unanimously defined. The chosen reference value represents a good average for terrestrial conditions. Although laboratory temperature might be somewhat greater than 290°K, this deviation is often not taken into consideration because the resulting error can usually be neglected. For instance, if the laboratory temperature is 310°K and the temperature of the equipment acting as the noise source is greater by 20°K than the ambient temperature, the error is about 0.6 dB.

Both the equivalent noise temperature and the noise figure are unambiguous noise characteristics of a block. The relation between them can be derived from equation (10.11.)

$$F = \frac{P_{n0}}{GP_{ni}}\Big|_{T_0} = \frac{GkB\left(T_0 + T_{red}\right)}{GkBT_0} = 1 + \frac{T_{red}}{T_0} \qquad (10.12)$$

or

$$T_{\mathrm{red}} = (F - 1)\cdot T_{\mathrm{o}} \qquad (10.13)$$

10.3. Noise Figure of Multistage Systems

It is a common task to determine the resulting noise figure of a system consisting of more than one block cascaded as shown on Fig. 10.1.

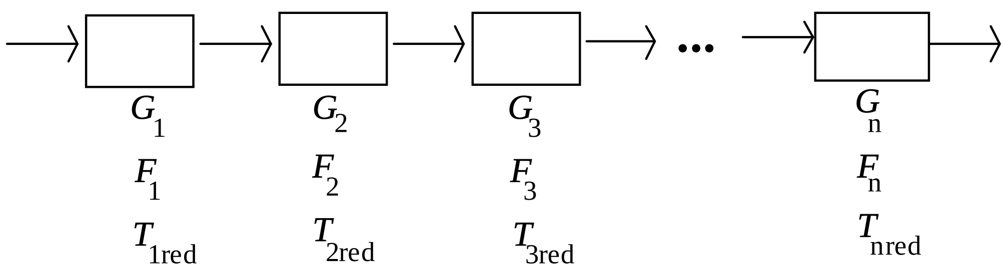

Figure 10.1. Parameters of a Multistage System

The resulting gain is the product of the individual gains

$$G = G_1\cdot G_2\cdot G_3 \ldots\cdot G_{\mathrm{n}} \qquad (10.14)$$

To compute the resulting equivalent noise temperature, the output noise power has to be determined first. For easier calculation, suppose that n = 2, i.e. there are only two stages. Output noise power of the first block is

$$P_{1\mathrm{o}} = G_1\cdot P_{\mathrm{ni}} + G_1\cdot B\cdot k\cdot T_{1\mathrm{red}} \qquad (10.15)$$

Since the output of the first stage acts as the input of the second one, using equation (10.10.) again

$$P_{2\mathrm{o}} = G_2\cdot P_{1\mathrm{o}} + G_2\cdot B\cdot k\cdot T_{2\mathrm{red}} \qquad (10.16)$$

Substituting (10.15.) into (10.16.)

$$P_{2o} = G_1 \cdot G_2 \cdot [P_{ni} + B \cdot k \cdot (T_{1red} + \frac{T_{2red}}{G_1})] \tag{10.17}$$

Resulting equivalent noise temperature can also be read out from the above formula

$$T_{red} = T_{1red} + \frac{T_{2red}}{G_1} \tag{10.18}$$

Results obtained for the two-stage case can be generalized for more stages as

$$T_{red} = T_{1red} + \frac{T_{2red}}{G_1} + \frac{T_{3red}}{G_1 G_2} + \ldots + \frac{T_{nred}}{G_1 G_2 \ldots G_{n-1}} \tag{10.19}$$

Using equation (10.12.), the resulting noise figure is

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots + \frac{F_n - 1}{G_1 G_2 \ldots G_{n-1}} \tag{10.20}$$

The last two equations point to an important conclusion: If the gains are sufficiently great, only the first stage is dominating from the point of the noise. Thus if the noise of a system has to be optimized, the parameters of the preamplifier are critical.

The performance of the preamplifier can, however, be remarkably degraded if a lossy block (e.g. a long cable) is inserted between the signal source (e.g. antenna) and the input, as shown in Fig 10.2.

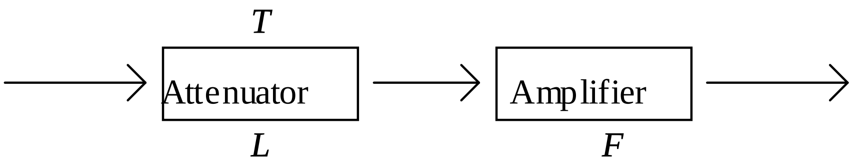

Figure 10.2. Two-Stage System with a Passive Input Block

Interpreting the attenuation L of the first stage as

$$G_1 = 1/L, \tag{10.21}$$

the relations obtained for the two-stage case can be applied.

To compute the noise figure and the equivalent noise temperature of the attenuator, let us complete the attenuator with an input block of the same temperature, as shown in Fig. 10.3.

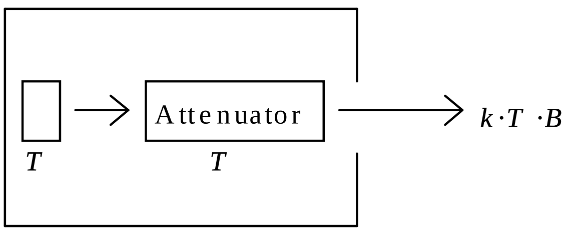

Figure 10.3. The Passive Block as a Source of Noise

The temperature of the resulting system is T, so that the output power in the radio range is $k \cdot T \cdot B$. On the other hand, the general rule defined by equation (10.10) can also be applied

$$k \cdot T \cdot B = \frac{1}{L} \cdot B \cdot k \cdot (T + T_{\mathrm{red}}) \qquad (10.22)$$

so that

$$T_{\mathrm{red}} = \mathrm{T} \cdot (L - 1) \qquad (10.23)$$

and

$$F_{\mathrm{L}} = 1 + \frac{T}{T_0} \cdot (L - 1) \qquad (10.24)$$

The above results are especially simple if the attenuator temperature is close to the reference. In this case

$$F_{\mathrm{L}} = L \qquad (10.25)$$

so the noise figure of the attenuator (cable) is the same as its attenuation (provided its temperature may be taken as 290°K). Furthermore, using equation (10.20), the resulting noise figure will be

$$F_{\mathrm{r}} = L \cdot F \qquad (10.26)$$

Since multiplication means addition of the corresponding values in dB, one may say that the noise figure is increased by as many dB-s by as many dB-s the signal has been attenuated before being amplified.

It was shown that if the noise temperature of a source (e.g. antenna) is equal to the reference temperature then the degradation of the signal-to-noise ratio is directly given by the attenuation. If the noise temperature of the source is lower, the signal-to-noise ratio can be degraded much more.

10.4. Effective Noise Temperature of Composite Sources

Let us examine a composite system consisting of a source and two attenuators each having different temperature as shown on Fig. 10.4.

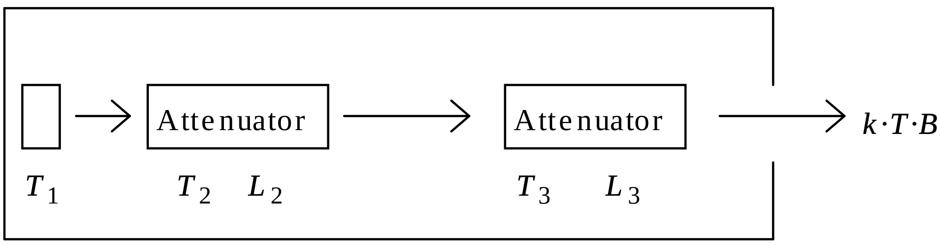

Figure 10.4. Composite Source of Noise

As the first step, a system consisting of only two elements will be discussed. The simplified system is shown on Fig. 10.5.

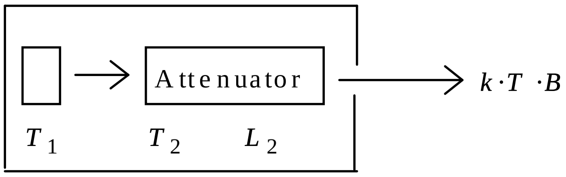

Figure 10.5. Complex System as the Source of Noise

According to equation (10.23), equivalent noise temperature at the attenuator input is $T_2(L_2$-1), the corresponding 'gain' is 1/L_2 so that

$$P = (T_1 + T_2(L_2 - 1))\, k{\cdot}B{\cdot}\frac{1}{L_2} \tag{10.27}$$

As it can be read out of the equation

$$T = T_1{\cdot}\frac{1}{L_2} + T_2(1 - \frac{1}{L_2}) \tag{10.28}$$

The above results can be applied for the system with two attenuators so that the resulting noise temperature of the complete system will be

$$T = T_1\,\frac{1}{L_2}\cdot\frac{1}{L_3} + T_2(1-\frac{1}{L_2})\cdot\frac{1}{L_3} + T_3(1-\frac{1}{L_3}) \tag{10.29}$$

For further discussion it is necessary to interpret the above result from the physical point of view. Let us examine the second member of the right side of the equation, as it seems to be a general one, preceded and followed by other terms.

We can see that the temperature is decreased, on one hand, because the following block (3) attenuates the noise generated by block 2 by a factor 1/L_3. On the other hand, the temperature is increased by a factor which is proportional to the inner loss of the block 2. It is important to mention that a ‘transparent’ block (i.e. one without loss) plays no role in the resulting loss, noise is generated only by passive elements.

As mentioned in chapter 10.2., the structure of a system is indifferent from the point of view of noise, the output power can always be computed from equation (10.1) if all its elements have the same temperature. For the time being we have used this equation for guided-wave structures but it is valid for systems containing radiating elements as well. Thus, if we measure noise power at the output of an antenna this will also satisfy equation (10.1), and in the radio range the simplified version (10.2) may be used.

From the physical aspect, the noise of such a system is not generated by the antenna because the lossless elements do not contribute to the resulting noise figure. The reason of the noise appearing at the antenna output is that there are noise sources in the radiation field picked up by the antenna (being weighted by the gain belonging to the corresponding direction). Weighted average temperature of differently located noise sources with different temperatures can be computed as

$$T_a = \frac{1}{4p}\cdot\iint_{(4p)} T(j\,,J)\cdot G(j\,,J)\,d\Omega \tag{10.30}$$

where T_a is the noise temperature of the antenna, $G(\varphi,\psi)$ is the direction-dependent gain of the antenna (referred to the izotrophic antenna) and dΩ is the infinitesimal range of the space angle.

The real conditions are even more complicated because of additional electric effects which cause considerable noise increase. Noise temperature of an antenna located on the Earth surface is determined by the following components:

- Cosmic background noise: 2.78 K, physical origin is unknown, according to the cosmological theory of the ‘big bang’, the cooled remains of the once hot universe might be the possible reason.
- Galaxian noise: radio radiation of our galaxy
- Tropospherical noise: radio noise caused by the atmosphere.
- Noise caused by Earth surface.
- Noise caused by the ‘near’ celestial bodies (Sun, Moon).
- Noise caused by the antenna loss.

10.5. Signal to Noise Balance of Radio Communications

As a restrictive factor, noise plays an important role in telecommunications, broadcasting, radio-astronomy, radar and navigation systems. The signal-to-noise ratio, i.e. the power of the useful signal compared to the noise power is an essential qualitative factor of systems. Generally, the amplification is also the function of frequency, thus the noise power is

$$P_n = \int_0^\infty G(f)\cdot S(f)\,\mathrm{d}f \tag{10.31}$$

Practically, spectral power density of the noise can be assumed as constant in the used frequency range (white noise), i.e.

$$S(f) = N_0 \tag{10.32}$$

Extracting the constant from the integral and introducing G_0 gain measured in the middle of the bandwidth

$$P_n = N_0\cdot G_0\cdot \frac{1}{G_0}\cdot\int_0^\infty G(f)\mathrm{d}f \tag{10.33}$$

On the right side, the term of the noise bandwidth

$$B = \frac{1}{G_0}\cdot\int_0^\infty G(f)\mathrm{d}f \tag{10.34}$$

has been introduced which can be used as if the gain were constant over the whole frequency range.

In the following, let us determine the signal-noise balance of a radio communication. We will concentrate only to the so-called RF signal-to-noise ratio neglecting the gain or the loss caused by the demodulator. Let us suppose the following quantities as given: P_t transmitting power, G_t gain of the transmitting antenna, G_r gain of the receiving antenna, D transmitter-receiver distance, T resulting noise temperature of the receiver, L additional attenuation, B noise bandwidth, and λ wavelength of the transmitter.

It was shown in Chapter 9 that free-space attenuation between two anisotrophic antennas is

$$\frac{P_t}{P_r} = \frac{16 \cdot p^2 \cdot D^2}{G_t \cdot G_r \cdot l^2} \tag{10.35}$$

Since the effective input noise can be expressed as $P_n = k \cdot T \cdot B$, the RF signal-to-noise ratio results in the following:

$$\left(\frac{S}{N}\right) = \frac{P_t \cdot G_t \cdot l^2}{16 \cdot p^2 \cdot D^2 \cdot L \cdot k \cdot B} \cdot \frac{G_r}{T} \tag{10.36}$$

It can be seen that signal-to-noise ratio is directly proportional to the factor G_r/T if all the other quantities are fixed. This factor is a basic parameter of a receiver station and it is usually given in dB/K (10 log of G_r/T).

10.6. Quantization Noise

From the physical point of view, information flow is not continuous but it is realized by elementary quanta of electrons or photons. If the signal is carried by the current then the minimal quantum is given by the charge of one electron, in the case of high frequency radiation the lower energy limit is $h \cdot f$.

In classical communication, thermal white noise played the most important role among the factors limiting sensitivity. Today's communication uses not only radio ranges but runs some five decades higher into the optical range which shows a rather quantized than wave character. Of course, the electromagnetic field, can neither be regarded as a continuous spatial wave or as a quantized flood of massive particles; both models reflect more or less just one side of the sides of the electromagnetic effect.

If the physical process of an information flow is quantized, it is a matter of chance how many elementary units will be observed over a selected time interval; the instantaneous value of the signal fixes only the expected value belonging to a certain number of elements. The actual number of elements varies accidentally around the expected value, the actual distribution can be described by the Poisson's distribution.

In communication systems, statistical fluctuation of incoming electrons or photons appears as noise. We are speaking about *quantum noise* when it is generated by photons while noise caused by current quantization is called *shot noise*. Since the physical basis of both the quantum noise and the shot noise is the same, quantum noise can be called shot noise of the photons. For the shot noise

$$\overline{i^2} = 2 \cdot B \cdot q \cdot I_0 \tag{10.37}$$

can be derived, where I_0 is the expected value of the current. Let us assume that the signal of a light source is received by an ideal demodulator, i.e. each incoming photon generates exactly one electron so that the output current of a signal having the power P is

$$I_0 = \frac{P \cdot q}{h \cdot f} \tag{10.38}$$

Interpreting the signal as a quantity proportional to the square of the expected value and taking the noise as the mean square of the fluctuation, signal-to-noise ratio can be written as

$$\left(\frac{S}{N}\right) = \frac{I_0^2}{\overline{i^2}} = \frac{P}{2 \cdot B \cdot h \cdot f} \qquad (10.39)$$

As it will be shown in the discussion of the modulation theory, baseband bandwidth B will be doubled around the carrier frequency, so that equation (10.39) might formally be interpreted as if the power spectral density of the quantum noise was

$$S(f) = h \cdot f \qquad (10.40)$$

Although equation (10.40) might have been derived even more exactly, one thing must not be forgotten. There is an essential physical difference between the quantum noise and the thermal noise: the later is present even if the signal is switched off while the quantum noise is generated only in the presence of the signal and because of its 'granular' nature, its replacement by a noise with constant power spectral density is only approximative.

Control Questions

1 How can the ‘optical’ range and the ‘radio’ range be distinguished?
2. How can a transmission system be characterized from the point of view of noise?
3. How can the resulting noise figure be determined?
4. What is the definition of the noise bandwidth?
5. What is the reason of the quantum noise?

Exercises

1. Compute the resulting noise figure of a system consisting of a cable and a preamplifier for both the cable or the preamplifier is connected first.
   Data: cable length is 15 m, specific attenuation of the cable 1 dB/m, gain of the preamplifier 20 dB and its noise figure 3 dB.
2. How much degraded is the noise temperature of an antenna if it is connected to the preamplifier through a cable which has 1 dB attenuation and temperature 290°K, and the original noise temperature of the antenna is 20°K? Give the value in dB.
3. How much increased is the noise temperature of an antenna with the beam angle of 5 degrees if the Moon appears in the received beam? Data of the Moon: temperature 300°K, diameter 3476 km, distance from the Earth 384000 km.

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[3] Morgan W.L., Gordon G.D.: Communication Satellite Handbook, John Wiley & Sons, 1989.