# 11. ANALOG MODULATION SCHEMES

## 11.1 Introduction

The general block diagram of an analog modulation system is shown in Fig. 11.1. The modulated signal $s(t)$ is generated by a modulator using the modulating source signal $s_m(t)$ to modulate a carrier of frequency $f_c$. The modulated signal is then passed through the channel where it is affected by different interferences and distortions (e.g. additive noise, linear and nonlinear distortion, etc.). The signal appearing at the channel output is demodulated and the resulting signal $s_d(t)$ is processed by the sink.

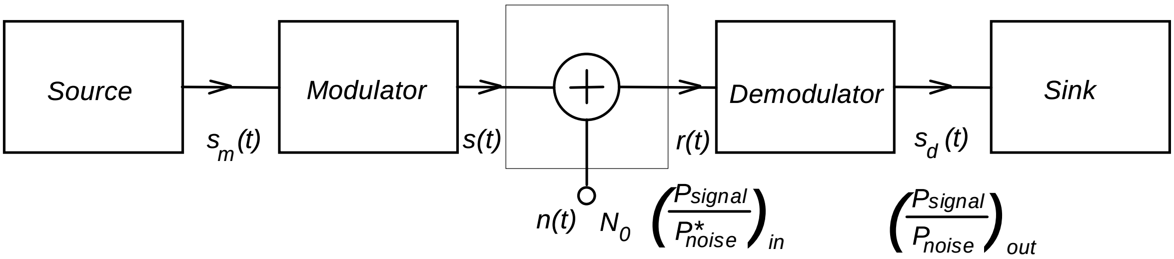


Figure 11.1 General Block Diagram of Analog Modulation Systems with Additive Gaussian Noise in the Channel

If the signal is affected only by additive Gaussian white noise then the system quality is characterized by the signal-to-noise ratio of the signal $s_d(t)$ which is defined as the ratio of the power of the useful signal to that of the noise. For the sake of simple comparison of different systems, let us define the signal-to-noise ratio at the demodulator input. Moreover, let it be defined so that it depends only on the power density of the signal and that of the noise but is not in direct relation to the total bandwidth of the modulated signal. For that purpose, let us introduce the so-called reference noise power: $P_n^* = 2 f_M \cdot N_o / 2 = f_M \cdot N_o$ where $f_M$ is the bandwidth of the modulating signal and $N_o$ is the single side power density of the Gaussian white-noise.

Since sinusoidal signals play a dominant role in analog modulation, let us start our discussion with systems which use a *sinewave* as the carrier. Some general questions have to be answered first and the possible solutions are then presented.

A modulated sinewave can be expressed in the following general form:

$$s_c(t) = a(t) \cdot \cos[\Theta(t)] \qquad (11.1)$$

where $s_c(t)$ is value of the modulated signal in time $t$, $a(t)$ is the instantaneous amplitude of the carrier, $\Theta(t)$ is the instantaneous phase of the carrier.

Since either the amplitude or the phase of the carrier -or both of them- may vary simultaneously, it is necessary to introduce the instantaneous values beside the time-average values normally used. The instantaneous value of the frequency ($f_i$) can then be defined as the time-derivative of the instantaneous phase:

$$f_i = \frac{1}{2\pi}\frac{d\Theta}{dt}, \text{ i.e. } \omega_i(t) = \frac{d}{dt}\left[\Theta(t)\right] \qquad (11.2)$$

It can be seen from equation (11.1) that either $a(t)$ or $\Theta(t)$ or both can be modulated by the source signal. *Amplitude modulation* and *angle modulation* are the terms used to distinguish which parameter is modulated:

| | | | |
|---|---|---|---|
| amplitude modulation: | $a(t) \neq$ const., | $f_i =$ const. | (11.3) |
| angle modulation: | $a(t) =$ const., | $f_i \neq$ const. | (11.4) |

In the following we review the different sorts of amplitude modulation and the most important angle modulation schemes.

## 11.2. Amplitude Modulation (AM)

As given by the name, the *amplitude* of the AM signals carries the information, i.e. the modulating signal is encoded somehow into the amplitude function $a(t)$. Let us see first, how an AM signal can be described in the time and in the frequency domain. Since $\omega_i$ is constant, $\Theta(t)$ can be obtained from (11.2) by simple integration:

$$\Theta(t) = \int_{-\infty}^{0} w_i \;\; \Theta(t) = \int_{-\infty}^{t} w_i \cdot ds \; = w_i \int_{0}^{t} ds + j \,, \quad w_i = 2p \cdot f_i , \tag{11.5}$$

where $\varphi$ is a constant which represents the phase in $t$=0. Since $f_i$ is constant, it is equal to the average carrier frequency, i.e.:

$$\Theta_{AM} = \omega_c t + \varphi \tag{11.6}$$

Substituting equation (11.6) into (11.1), the general expression of the AM signal is

$$s_{AM}(t) = a(t) \cdot \cos\left[\Theta(t)\right] = a(t) \cdot \cos\left[\omega_c t + \varphi\right] \tag{11.7}$$

Since the initial phase of an AM signal is usually indifferent, let us simply suppose that $\varphi = 0$, thus

$$s_{AM}(t) = a(t) \cdot \cos(\omega_c t) \tag{11.8}$$

which is the general time-domain representation of AM signals.

Suppose that the amplitude function $a(t)$ containing the modulating signal is a band-limited signal with the highest frequency $f_M$, i.e. the spectrum of the $a(t)$ function extends from ($-f_M$) to ($+f_M$) in the complex frequency domain (see part (a) of Fig. 11.2.)

Let $A(f)$ be the Fourier transform (spectrum) of $a(t)$ and let us examine how it is influenced by modulation. The Fourier transform of the modulated signal, $S_{AM}(f)$, can be written as follows:

$$\begin{aligned} S_{AM}(f) &= \int_{-\infty}^{-\infty} s_{AM}(t) \cdot e^{-jw \cdot t} \cdot dt = \int_{-\infty}^{+\infty} a(t) \cdot \cos(w_c t) \cdot e^{-jw \cdot t} \cdot dt = \\ &= \frac{1}{2} \int_{-\infty}^{+\infty} a(t) \cdot e^{-j(w - w_c)t} \cdot dt + \frac{1}{2} \int_{-\infty}^{+\infty} a(t) \cdot e^{-j(w + w_c)t} \cdot dt. \end{aligned} \tag{11.9}$$

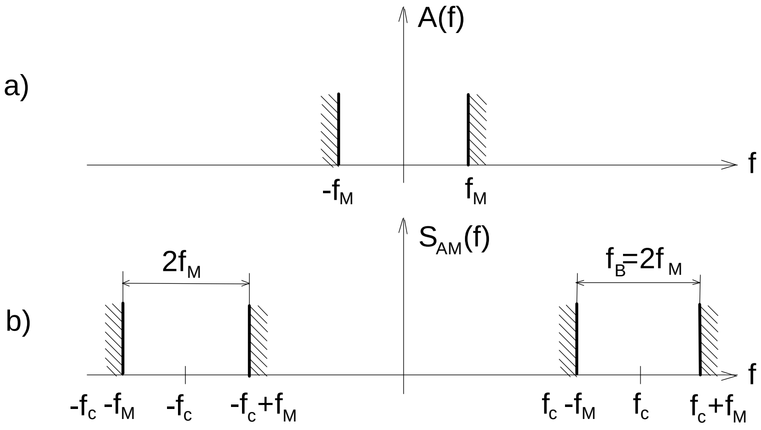


Figure 11.2. The Amplitude Function and the Modulated Signal in the Frequency Domain

Two integral expressions of equation (11.9.) can be considered as if the spectrum $A(f)$ had been shifted along the frequency axis to $(+f_c)$ and $(-f_c)$ and its amplitude decreased by half of the original:

$$s_{AM}(f) = \frac{1}{2}A(f - f_c) + \frac{1}{2}A(f + f_c) \qquad (11.10)$$

The graphic form of equation (11.10) is presented in part b) of Fig. 11.2. It is important to notice that AM is a *linear modulation* since the shape of the spectrum has been affected by linear operations, i.e. it has been shifted to $(-f_c)$ and $(+f_c)$ and multiplied by 0.5. It can be also seen that to avoid the spectrum aliasing, the carrier frequency must be at least the double of the maximum modulating frequency.

*11.2.1. Sinewave Modulated AM Signals*

Further properties of the AM modulation will be examined by using a simple cosine waveform as the modulating signal $s_m(t)$:

$$s_m(t) = U_m \cdot \cos(\omega_m t) \qquad (11.11)$$

To transmit the above signal by AM, the $a(t)$ has to include somehow the modulating signal $s_m(t)$. At first sight it seems obvious to make $s_m(t)$ equal with $a(t)$. Because of a practical reason let us choose, however, a more general relation:

$$a(t) \stackrel{\Delta}{=} U_c + s_m(t) \qquad (11.12)$$

where $U_c$ is constant and represents the amplitude of the unmodulated carrier (when $s_m(t) \equiv 0$). By substituting (11.11) into (11.12)

$$a(t) = U_c + U_m \cdot \cos(\omega_m t) \qquad (11.13)$$

The simple form of the modulating signal enables us to examine the shape of the AM signal for different ratios of amplitudes $U_c$ and $U_m$. Starting with equation (11.8), the time function of the AM signal is

$$s_{AM}(t) = a(t) \cdot \cos(\omega_c t) = \left(U_c + U_m \cdot \cos(\omega_m t)\right) \cdot \cos(\omega_c t) =$$

$$= U_c \cdot \cos(\omega_c t) + U_m \cdot \cos(\omega_m t) \cdot \cos(\omega_c t) \qquad (11.14)$$

which can be rewritten as

$$s_{AM}(t) = U_c \cdot \cos(\omega_c t) + \frac{U_m}{2} \cos[(\omega_c + \omega_m)\, t] + \frac{U_m}{2} \cos[(\omega_c - \omega_m)\, t] \qquad (11.15)$$

The last equation is suitable to present the three basic types of amplitude modulation:

- AM-DSB (Double Sideband Amplitude Modulation): All the three components of equation (11.15) are present in the signal. The bandwidth of this modulation is $f_B = 2 \cdot f_M$, its characteristic term is the *modulation depth* defined as $m_a = U_m/U_c$ which can change between 0 and 1. The vector diagram and the time and frequency representation of the AM-DSB signal are shown in Fig 11.3.

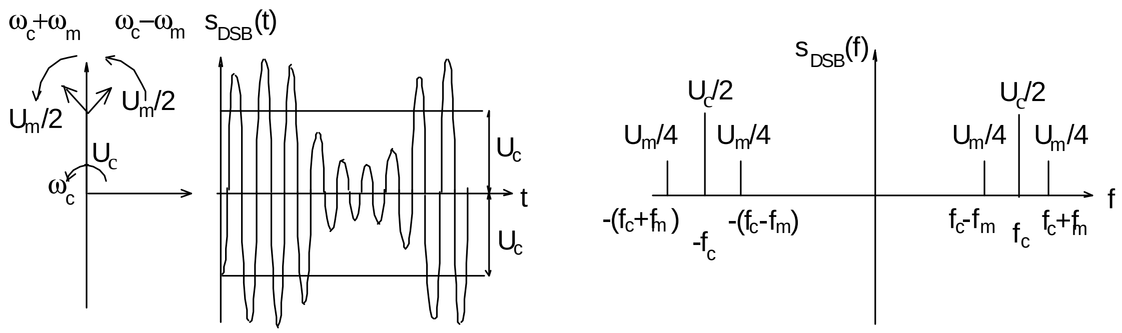


Figure 11.3. Vectorial Diagram and Time and Frequency Domain Representations of the AM-DSB Signal

- AM-DSB/SC (Double Sideband/Suppressed Carrier Amplitude Modulation): The first member of equation (11.15) is eliminated (e.g. suppressed by a filter or by a balanced multiplier), i.e. carrier frequency is absent in the modulated signal. The bandwidth of the AM-DSB/SC signal is $f_B = 2 \cdot f_M$, the vector diagram and the time and frequency representation are shown in Fig 11.4.

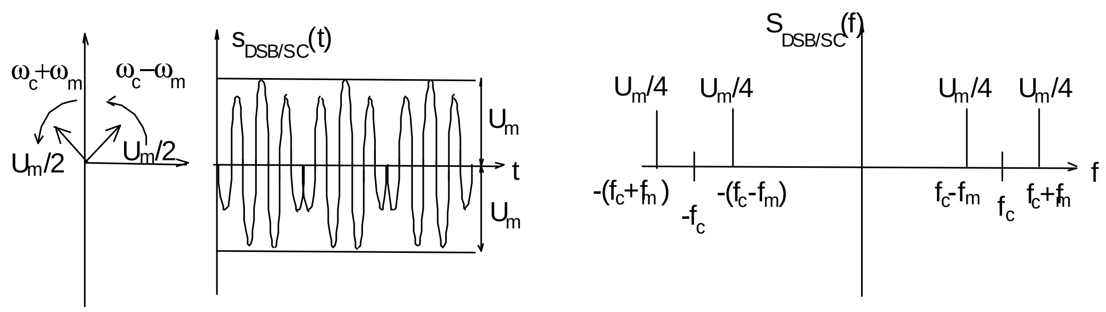


Figure 11.4. Vectorial Diagram and Time and Frequency Domain Representations of the AM-DSB/SC Signal

- AM-SSB/SC (Single Sideband/Suppressed Carrier Amplitude Modulation): Here the first and the second (or the third) member of equation (11.15) is zero thus only components above (or under) the carrier frequency appear in the modulated signal. The bandwidth of the SSB signal is $f_B = f_M$, the vectorial diagram and the time and frequency representation are shown in Fig 11.5.

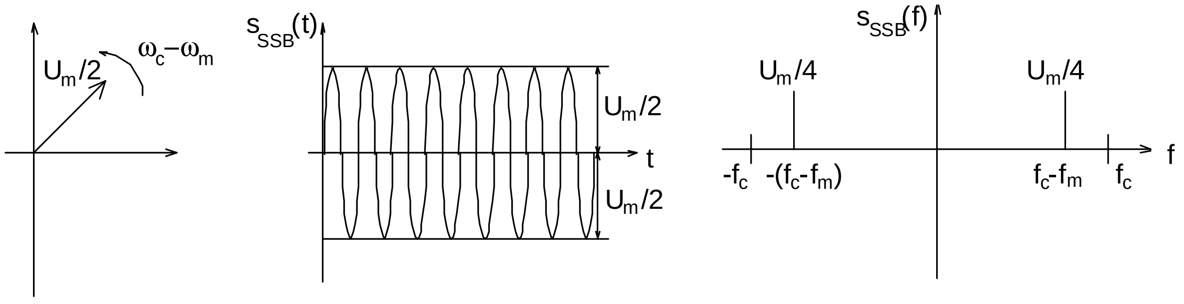


Figure 11.5. Vectorial Diagram and Time and Frequency Domain Representations of an AM-SSB/SC Signal

### *11.2.2. AM Signal Demodulation*

AM signals are generally demodulated by product detectors (multipliers). AM-DSB is an exception since it can be demodulated also by the so-called envelope detector. In the following, the demodulation by multipliers is discussed.

Let us examine what the output of an ideal multiplier will be if one of its inputs is driven by an AM signal and the other by a sinewave of the same frequency as the carrier, shifted by φ in phase. The product of the two signals denoted as $s_d(t)$ is as follows:

$$\begin{aligned} s_d(t) &= s_{AM}(t)\cdot\cos(w_c\cdot t + j) = a(t)\cdot\cos(w_c\cdot t)\cdot\cos(w_c\cdot t + j) = \\ &= \frac{a(t)}{2}\cos(j) + \frac{a(t)}{2}\cos(2w_c\cdot t + j). \end{aligned} \qquad (11.16)$$

Suppressing the second member of the sum by a filter, the desired baseband signal $a(t)$ is obtained almost exactly (regardless of the 0.5 factor and provided that φ = 0).

Let us determine the S/N ratio at the demodulator output if the signal has been passed through an additive noisy channel. The signal at the demodulator input is

$$r(\mathrm{t}) = s_{AM}(\mathrm{t}) + n^*(\mathrm{t}) \qquad (11.17)$$

where $s_{AM}(t)$ is the AM signal, $n^*(t)$ is that part of the Gaussian white-noise $n(t)$ with $N_0/2$ double-side power-density which falls into the range of the useful signal.

Since the bandwidth of an AM-DSB signal is $4\cdot f_M$, the entire power of the $n(t)$ is $P_N = 4\cdot f_M\cdot N_0/2$. It is known that $n(t)$ can be decomposed into modulation form as

$$n^*(t) = n_c^*(t)\cdot\cos(\omega_c t) + n_s^*(t)\cdot\sin(\omega_c t), \qquad (11.18)$$

where $n_c^*(t)$, $n_s^*(t)$ is the independent baseband Gaussian noise pair with double side power density $N_0$ and bandwidth $f_M$.

Provided the demodulator works under noisy conditions also according to equation (11.16), then (if φ = 0):

$$\begin{aligned} s_d(t) &= r(t)\cdot\cos(\omega_c t) = a(t)\cdot\cos^2(\omega_c t) + n_c^*(t)\cos^2(\omega_c t) + \\ &+ n_s^*(t)\cdot\sin(\omega_c t)\cdot\cos(\omega_c t) \overset{D}{=} \frac{a(t)}{2} + \frac{n_c^*(t)}{2}, \end{aligned} \qquad (11.19)$$

where $\Delta$ denotes the baseband part of the signal. Introducing the input reference signal-to-noise ratio:

$$\left(\frac{P_S}{P_N^*}\right)_{in} = \frac{\frac{\mathrm{M}\left\{a^2(t)\right\}}{2}}{f_M N_o} = \frac{\mathrm{M}\left\{a^2(t)\right\}}{2 f_M N_o} \tag{11.20}$$

while at the demodulator output

$$\left(\frac{P_S}{P_N}\right)_{out} = \frac{\frac{1}{4}\mathrm{M}\left\{a^2(t)\right\}}{\frac{1}{4}\mathrm{M}\left\{n_o^{*2}(t)\right\}} = \frac{\mathrm{M}\left\{a^2(t)\right\}}{2 f_M N_o}, \tag{11.21}$$

which means that in the case of AM-DSB the input reference S/N ratio is the same as the output S/N ratio. Obviously, this is true only if the entire power of the $a(t)$ function carries information (as it is the case with the AM-DSB/SC). Normally, the AM-DSB uses only a part of the amplitude function $a(t)$ (see eq. 11.6), thus the output S/N ratio decreases if the modulation depth is reduced.

Let us note that for AM-SSB/SC signal the input reference and the output S/N ratios of the demodulator are equal:

$$\left(\frac{P_S}{P_N}\right)_{out} = \left(\frac{P_S}{P_N^*}\right)_{in} = \frac{\frac{\mathrm{M}\left\{a^2(t)\right\}}{4}}{f_M N_o} = \frac{\mathrm{M}\left\{a^2(t)\right\}}{4 f_M N_o} \tag{11.22}$$

which means that S/N ratio of AM-SSB/SC is only half that of the AM-DSB.

## 11.3 Angle Modulation

As earlier defined, in the case of angle modulation the amplitude of the carrier is constant while the instantaneous frequency -and so the instantaneous phase- is changing with the modulating signal (see eq. 11.4). Similarly as for the AM systems, the relation between the modulating signal $s_m(t)$ and the frequency (or the phase) of the modulated signal has to be determined first. Obviously, the simpler the relation is, the easier it is to modulate and to demodulate the signal. Since the linear relation is the simplest, two types of angle modulations are used: *frequency modulation* and *phase modulation*. The frequency modulation (FM) is defined by

$$f_i = \frac{1}{2\pi}\frac{d\Theta}{dt} = k_{FM}\cdot s_m(t) + f_c \tag{11.23}$$

while the phase modulation (PM) is defined by

$$\Theta(t) = k_{PM}\cdot s_m(t) + \omega_c t \tag{11.24}$$

where $k_{FM}$ and $k_{PM}$ are constants with different units and $f_c$ is the frequency of the unmodulated carrier (also constant). Using equations (11.1) and (11.24), the general form of the FM signal is as follows:

$$s_{FM}(t) = a(t)\cdot\cos(\Theta(t)) = U_c \cos\left[2p\left(f_c \cdot t + k_{FM}\int_0^t s_m(s)ds\right)\right] =$$

$$= U_c \cos\left(w_c \cdot t + 2pk_{FM}\int_0^t s_m(s)ds\right)$$

while the same for the PM signal:

$$s_{PM}(t) = a(t)\cdot\cos(\Theta(t)) = U_c\cdot\cos\big(\omega_c\, t+k_{PM}s_m(t)\big) \tag{11.25}$$

Instead of the general form of the modulating signal,

$$s_m(t) = U_m\cdot\cos(\omega_m\, t) \tag{11.26}$$

will be used for further discussion, similarly as for the AM. Substituting (11.26) into (11.25):

$$s_{FM}(t) = U_c \cos\left(w_v t + 2p\cdot k_{FM}\int_0^t U_m \cos(w_m s)\mathrm{d}s\right) =$$

$$= U_c\cos\left[\ \omega_v t + \frac{k_{FM}\, 2\pi U_m}{\omega_m}\sin(\omega_m t)\ \right]$$

$$= U_c\cos\left[\ \omega_v t + \frac{k_{FM} U_m}{\mathrm{f}_m}\sin(\omega_m t)\ \right]$$

$$s_{PM}(t) = U_c\cdot\cos\big(\omega_v t+k_{PM}U_m\cdot\cos(\omega_m t)\big) \tag{11.27}$$

So the information represented by the modulating signal is encoded into the FM signal in the form of frequency changes of the carrier around a central value ($f_v$). The amplitude of the modulating signal corresponds to the maximum difference or the *deviation* of the carrier frequency from $f_v$ while the frequency of the modulating signal is equal to the frequency the instantaneous carrier frequency is changing around the average $f_v$. Let us denote the maximum frequency deviation as $f_D$, i.e.:

$$f_D = k_{FM}\cdot U_m \tag{11.28}$$

and then substituting (11.26) into (11.23):

$$f_p=k_F\cdot s_m(t)+f_c=k_{FM}\cdot U_m\cos(\omega_m t) + f_c = f_c+f_D\cdot\cos(\omega_m t). \tag{11.29}$$

which shows that $f_D$ is the *maximum* deviation from the unmodulated center frequency $f_c$. (To distinguish the two deviations, the instantaneous value is denoted as $f_d$).

Similarly to the definition of the $m_a$, the ratio $k_{FM}\cdot U_m/f_m$ in equation (11.27) is called the FM modulation factor and is denoted as $m_f$, i.e.:

$$M_f = \frac{k_{FM} U_m}{\mathrm{f}_m} = \frac{f_D}{f_m} \tag{11.30}$$

and the product $k_{PM}\cdot U_m$ is called the PM modulation factor and is denoted as $m_p$, i.e.:

$$m_p = k_{PM}\cdot U_m \tag{11.31}$$

Both the $m_f$ and the $m_p$ have clear physical meanings which can be read out from equation

(11.27): they represent the maximum phase deviation of the modulated carrier with respect to the phase of the unmodulated one. For that reason they are also called phase deviations. The time-domain waveform of an FM signal modulated by a sinewave is shown in Fig. 11.6.

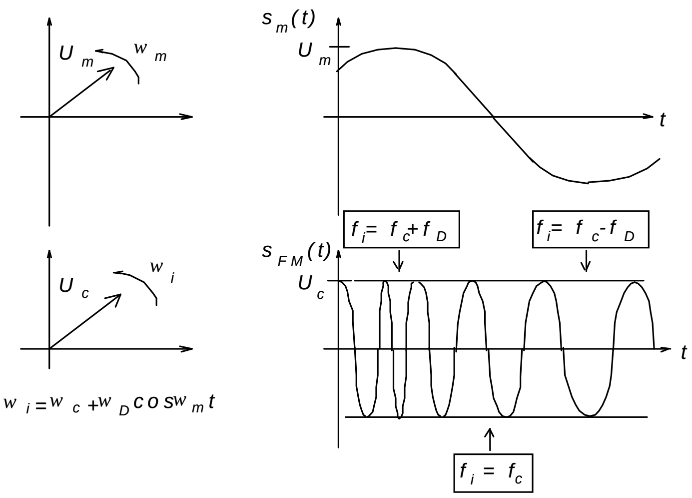


Figure 11.6 Time Domain Representations of the Modulating and the FM Signal

It can be shown by a detailed analysis that the bandwidth of an FM signal is: $f_B = 2 \cdot a \cdot f_m$ where

$$a = \begin{cases} \cong 1 & \text{if } m_f \langle\ 0{,}1 \\ \cong m_f & \text{if } m_f \rangle\ 10 \text{ and} \\ \cong 1 + m_f + \sqrt{m_f} & \text{otherwise} \end{cases} \qquad (11.32)$$

and $f_m$ is the frequency of the modulating sinewave.

*11.3.1. FM Signal Demodulation*

To demodulate an FM signal, a circuit with output voltage proportional to the instantaneous frequency of the input signal is needed. The amplitude response of the ideal FM demodulator is shown in Fig. 11.7. Circuits with such characteristics are called *frequency discriminators.*

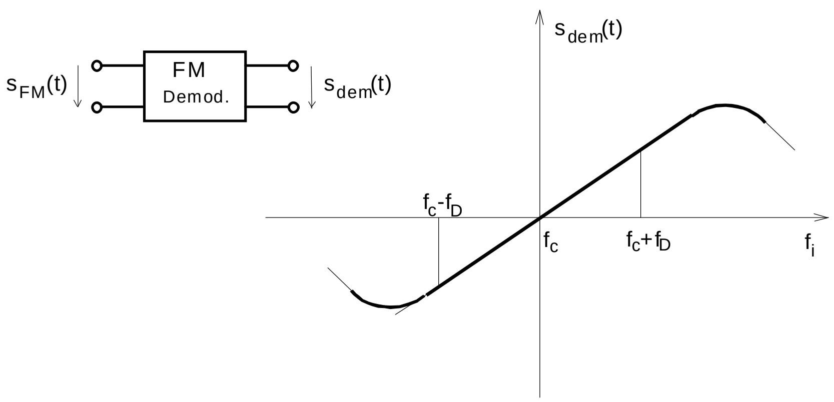


Figure 11.7. The Ideal FM Demodulator

Suppose the input of a frequency discriminator is driven by an FM signal given by equation (11.25):

$$s_{FM}(t) = U_c \cos\left[w_c t + 2p \cdot k_{FM} \int_0^t s_m(s)ds\right] \tag{11.33}$$

If the discriminator is ideal, the output signal will be

$$s_{dem}(t) = k_{discr} \cdot 2\pi f_i = k_{disc} \cdot 2\pi\left(f_c + k_{FM} \cdot s_m(t)\right) \tag{11.34}$$

since the instantaneous frequency is determined by the time-derivative of the argument of the cosine function ($\Theta(t)$).

The ideal discriminator can be approximated by deriving the signal and then demodulating the output by an envelope detector. Namely, if the FM signal is derived in time

$$\frac{d}{dt}\left[s_{FM}(t)\right] = U_c \cdot 2p \cdot \left(f_c + k_{FM} \cdot s_m(t)\right) \cdot \sin\left[2p\left(f_c \cdot t + k_{FM} \int_0^t s_m(s)ds\right)\right] \tag{11.35}$$

the result is an FM signal, the *amplitude* of which changes proportionally to the modulating signal $s_m(t)$. Demodulation of this AM-FM signal by an envelope detector will lead to a voltage proportional with the amplitude, i.e. with the modulating signal.

If noise is also present, then the S/N ratio at the demodulator input can be determined as follows:

$$\left(\frac{P_s}{P_n^*}\right)_{in} = \frac{U_c^2 / 2}{f_M \cdot N_o} = \frac{U_c^2}{2 f_M \cdot N_o} \tag{11.36}$$

The S/N ratio at the demodulator output can be calculated for low-level noise as follows: From equation (11.34), power of the useful signal -provided that $k_d$=1- is obtained from the following formula:

$$P_s\Big|_{out} = (2p)^2 k_{FM}^2 \cdot M[s_M^2(t)] \tag{11.37}$$

Suppose that a band-limited Gaussian white noise (see eq. 11.18) is added the modulated signal (eq. 11.33). The sinusoidal baseband component of this noise ($n_s^*(t)$) produces a phase-noise or the so-called *jitter* which can be defined by

$$e(t) = \frac{n_s^*(t)}{U_c} \quad \text{if } |\varepsilon(t)| << 1 \tag{11.38}$$

This 'phase' can be determined by computing the phase change caused by the noise given by equation (11.25). The derivative of such 'phase-noise' is then added to the useful signal and the resulting sum can be considered as the instantaneous 'frequency-noise':

$$\frac{d}{dt}\left[e(t)\right] = \frac{1}{U_c} \frac{d}{dt}\left[n_s^*(t)\right] \tag{11.39}$$

The entire baseband noise power in the frequency range $f_M$ is given by the following expression:

$$P_n\Big|_{out} = \frac{1}{U_c^2}\int_{-w_M}^{+w_M}\frac{N_o}{2p}w^2dw = \frac{N_o}{2p}\frac{2}{U_c^2}\frac{w_M^3}{3} = (2p)^2\frac{2}{3U_c^2}N_o f_M^3 \qquad (11.40)$$

so that for the output S/N ratio

$$\left(\frac{P_s}{P_n}\right)_{out} = \frac{(2p)^2 k_{FM}^2 \cdot M\left[s_m^2(t)\right]}{(2p)^2\dfrac{2}{3U_c^2}N_o f_M^3} = 3\frac{k_{FM}^2 \cdot M\left[s_m^2(t)\right]}{f_M^2}\cdot\frac{U_c^2}{2f_M N_o} \qquad (11.41)$$

is obtained. It can be seen from equation (11.41) that the S/N ratio at the demodulator output is

$$\left(\frac{P_s}{P_n}\right)_{out} = 3\cdot\left(\frac{P_s}{P_n^*}\right)_{in}\frac{k_{FM}^2 \cdot M\left[s_m^2(t)\right]}{f_M^2}, \qquad (11.42)$$

where $k_{FM}^2 \cdot \{s_m^2(t)\}$ is the square of the frequency deviation (see eq. (11.28)). So if the radio frequency power is kept constant and the noise level is small, S/N ratio can be improved by increasing the frequency deviation of the FM.

## Control questions

1. Give the general structure of analog modulation systems and define the amplitude and the angle modulation.
2. Draw the vector diagram, the time function and the spectrum of the AM-DSB, AM-DSB/SC and AM-SSB/SC signals in the case of sinusoidal modulation.
3. What is the reference noise power?
4. Determine the signal-to-noise ratio of the AM-DSB modulation system for the Gaussian white-noise of one sided power density $N_o$.
5. What is the frequency and the phase deviation and how is the modulation factor defined in FM and PM systems?
6. How can the bandwidth of an FM signal modulated by a sinewave be approximately computed?
7. Determine the signal-to-noise ratio of the FM modulation system for the Gaussian white-noise of one sided power density $N_o$.

## Exercises

1. Draw the spectrum of an AM-DSB signal ($f_c$ = 20 kHz), if

   (a) the time function of the modulating signal is
   $s_m(t) = U_{m1}\cos(\omega_1 t) + U_{m2}\cos(\omega_2 t)$,
   $U_c$ = 1V, $U_{m1}$ = 0.2V, $U_{m2}$ = 0.5 V,
   $\omega_1 = 2\cdot\pi 10^3$ rad/sec, $\omega_2 = 2\cdot\pi\cdot 2\cdot 10^3$ rad/sec

   (b) the Fourier-transform of the modulating signal is

$$S_m(f) = \begin{cases} C\left(1-\dfrac{|f|}{f_M}\right) \text{ if} |f| \le f_M, \\ 0 \text{ otherwise} \end{cases}$$

$U_c = 1$ V ; $C = 0.5 \cdot 10^{-3} \left[\dfrac{V}{Hz}\right]$; $f_M = 1$ kHz

2. Draw the spectrum of the AM-DSB/SC and of the AM-SSB/SC signal, if
   (a) the time function of the modulating signal is

   $s_m(t) = U_{m1}\cos(\omega_1 t) + U_{m2}\cos(\omega_2 t)$

   (b) the spectrum of the modulating signal is

   $$S_m(f) = S_m(f) = \begin{cases} C\left(1-\dfrac{|f|}{f_M}\right); \text{ if} |f| \le f_M, \\ 0 ;\text{otherwise,} \end{cases}$$

   and the data are the same as in Exercise 1.

3. Determine the output signal-to-noise ratio of the AM-DSB if the data are as follows:

   (a) $U_c = 1$ V; (b) $U_c = 0$ V
   $f_M = 3$ kHz; $N_0 = 10^{-6} \left[\dfrac{W}{Hz}\right]$;
   $a(t) = U_v + U_m\cos(\omega_m t)$; $U_m = 0.5$ V,
   The reference resistance is 1 Ω.

4. What are the values of $\omega_D$ and $m_p$ of a PM system if

   $s_{PM}(t) = U_c\cos(\omega_c t + cU_m\cos(\omega_m t))$
   $U_c = 1$ V; $\omega_c = 2 \cdot 10^6 \dfrac{rad}{\sec}$ ; $c = 0{,}1\dfrac{1}{V}$
   $U_m = 1$ V; $\omega_m = 2 \cdot 10^3 \dfrac{rad}{\sec}$

5. What is the maximum phase deviation and $m_f$ of an FM system if

   $s_{FM}(t) = U_c\cos(\omega_c t + c \cdot U_m \sin(\omega_m t))$
   $U_c = 1$ V; $\omega_c = 2 \cdot 10^6 \dfrac{rad}{\sec}$; $c = 0{,}2 \dfrac{1}{V}$
   $U_c = 1$ V; $\omega_c = 2 \cdot 10^2 \dfrac{rad}{\sec}$

6. What is the approximate value of the bandwidth of an FM system if its parameters are as follows:

   $s_{FM}(t) = U_V\cos\left(\omega_V t + \dfrac{k_{FM}U_m}{f_m}\sin(\omega_m t)\right)$; $k_{FM} = 10^3 \dfrac{Hz}{V}$; $U_m = 1$ V
   (a) $f_m = 10^3$ Hz; (b) $f_m = 10$ Hz

7. Draw the ratio of the bandwidth of a sinewave-modulated FM signal to the frequency of the modulating signal as the function of $m_f$.

8. Compute the output signal-to-noise ratio of a sinewave-modulated FM system if the data are as follows:

$$s_m(t) = U_m \cos(\omega_m t);$$
$$U_c = 1\text{ V} ;\ f_m = f_M = 10^3\text{ Hz} ;\ N_0 = 10^{-5} \frac{W}{Hz}$$
$$U_m = 1\text{ V} ;\ k_{FM} = 10^3 \frac{Hz}{V}$$ ; The reference resistance is 1 Ω.

## References


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