1th session - Basic signal processing

Exercise 1.1: Let us calculate the "crest factor" of a signal with amplitude A and frequency f_0 if the signal shape is:

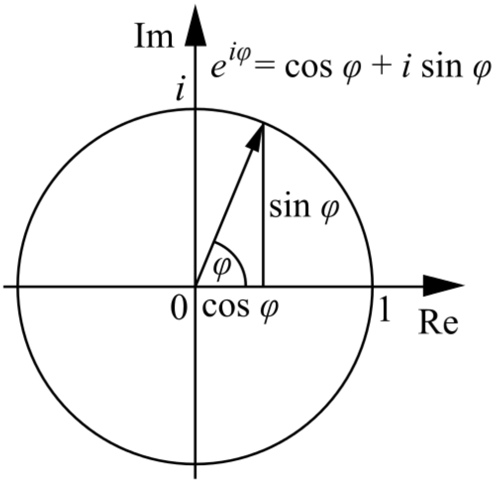
a) sine wave

b) symmetric square.

(Help:

$$C = \frac{|X|_{peak}}{X_{rms}}, \quad X_{rms} = \sqrt{\frac{1}{T}\int_{t=0}^{T} X^2(t)dt}, \quad P_{mean} = \left.\frac{X_{rms}^2}{R}\right|_{for R=1\Omega} = X_{rms}^2$$

Euler(1707-1783): "the Most Beautiful Mathematical Formula Ever", $e^{j\pi}+1=0$, where $j \equiv i = \sqrt{-1}$



Corollary: $\cos x \cos y = \frac{1}{2}(\cos(x+y)+\cos(x-y))$

Peak values: A, rms values: A/$\sqrt{2}$*(sine) and A (square). Crest factors:*$\sqrt{2}$*(sine) 1 (square)*)

Exercise 1.2: Let us calculate the "crest factor" of the sum of two sine wave signal, both with amplitude A, - one with frequency f_1, the other with f_2 .

(Help:

Generally - but not always! - peak values sum up. The same with powers.

C= (2A)/sqrt(A^2*/2+A*^{2}*/2)=2)*

Exercise 1.3: Let us determine the crest factors of the two signals below.

$$x_1(t) = A\cos(2\pi\cdot f_0 t) + A\cos(2\pi\cdot 2 f_0 t) + A\cos(2\pi\cdot 3 f_0 t)$$
$$x_2(t) = A\cos(2\pi\cdot f_0 t) + A\sin(2\pi\cdot 2 f_0 t) + A\cos(2\pi\cdot 3 f_0 t)$$

(Help:
The power of the two signals are the same, and the rms value is $A\sqrt{3/2}$.

The peak value of the 1st signal is obviously *3A*.

The peak value of the 2nd signal is not so easy to calculate...

One possible way is:

$$x_2(t) = A\sin(2\pi\cdot 2 f_0 t) + 2\cdot A\cos(2\pi\cdot\frac{3-1}{2} f_0 t)\cos(2\pi\cdot\frac{3+1}{2} f_0 t)$$

from which:

$$|x_2(t)| \le \sqrt{A^2 + 4A^2\cos^2(2\pi\cdot f_0 t)} \le A\sqrt{5}$$

First signal: $\sqrt{6}$, *second:* < $\sqrt{10/3}$.)

Exercise 1.4: *Let us compare the ideal and and first order low pass filters!*

Transfer functions:

ideal low pass filter with bandwidth B	first order low pass filter with bandwidth B
$H(f) = e^{-j2\pi fT}$, if $\|f\| < B$ else $H(f) = 0$ (T is the latency of the filter)	$H(f) = \frac{1}{1 + j\cdot f/B}$

The impulse response of the ideal filter:

$$h(t) = \int_{-\infty}^{\infty} H(f)\cdot e^{j2\pi ft}df = \int_{-B}^{B} 1\cdot e^{-j2\pi fT}\cdot e^{j2\pi ft}df = \left.\frac{e^{j2\pi f(t-T)}}{j2\pi(t-T)}\right|_{-B}^{B} = \frac{2\sin\left(2\pi B(t-T)\right)}{2\pi(t-T)}.$$

The impulse response of the first order filter. Help: the function is an exponential function.

$$H(f) = \int_0^{\infty} a\cdot e^{-bt}\cdot e^{-j2\pi ft}dt = a\cdot\int_0^{\infty} e^{-(b+j2\pi f)t)}dt = a\cdot\left.\frac{-e^{-(b+j2\pi f)t}}{b + j2\pi f}\right|_0^{\infty} = \frac{a/b}{1 + j\cdot 2\pi f/b}$$

Parameters: $a = b = 2\pi B$.

2th session - sampling and quantization

Exercise 2.1: A sampling-reconstruction (ADC-DAC) system is operating with a sampling frequency of 8kHz. The input (anti-aliasing) and output filter characteristics are identical with the following parameters:

$$H_I(f) = H_O(f) = \begin{cases} 1 & if \quad |f| \le 3\,kHz \\ 2.5 - |f/2| & if \quad 3kHz < |f| \le 5kHz \\ 0.01 & if \quad 5kHz < |f| \le 10kHz \\ 0 & else \end{cases}$$

A) If an input sine wave signal with an amplitude of 2 *V* and frequency of 1 kHz is given, then a spectral component identical with the input can be measured in the output. However, other spectral components can be measured as well. Let us calculate them (amplitudes, frequencies).
B) What would be the output if the characteristics of the input signal are the following: sine wave, 2V, 4.5 kHz?

(The effect of quantization is not considered here.)

Exercise 2.2: Choosing the appropriate sampling frequency

The spectral density function of a stationary (stochastic) signal is generally zero except in the range of 0-3 *kHz* and 7-8 *kHz* bands, where its value is constant.

a) Let us calculate the spectral density function in the negative frequency domain.
b) Calculate the sampling frequency range(s) appropriate for the signal to be completely reconstructed from samples.
c) Give the output (signal reconstruction) filter characteristics according to the previous sampling frequencies.

Exercise 2.3:
A real signal contains measurable spectral components only in the frequency range of 19 *kHz* and 25 *kHz*. This signal is digitized and processed further with a digital signal processor (DSP), then the (modified) signal is reconstructed from the samples.

A) What is the smallest sampling frequency with which the - theoretically - perfect D/A conversion can be performed? (The effect of quantization is not yet considered.)
B) Considering the effect of quantization, what would be the improvement in terms of SNR [dB] if a sampling frequency of 50kHz would be applied instead of the previously calculated minimal one?

(Help: $SNR = \frac{U_p^2/2}{2B\Delta^2/(12 f_s)} = \frac{3}{c^2} \cdot \frac{f_s}{2B} \cdot 2^{2n}$)

Exercise 2.4:
A 15 *kHz* bandwidth signal is sampled with a frequency of 44 *kHz* and the samples are represented by 20 bits codewords. At the reconstruction phase, the D/A converter is a 16 bit one, and the most significant bits are kept. By default, the output (reconstruction) filter can be considered as an ideal low-pass one.

a) What can be the improvement in terms of SNR [dB] if the sample frequency is increased to 4 times higher than the original one through sample interpolation?
b) What kind of spectral "side effect" can be experienced if the output filter is replaced by a first order low-pass filter?

3rd session - Radio waves

Exercise 3.1: Open air wave propagation (no reflections)

Let us calculate the attenuation of the open air radio link with the following conditions. The distance between the transmitter and receiver is 10 *km*. The carrier frequency is 450 MHz, and the gain of both the transmitter and receiver antennas is 20 *dB*.

(Help: $a_{open\_air} = 10 \cdot \lg\left(\frac{P_T}{P_R}\right) = 20 \cdot \lg\left(\frac{4\pi r}{\lambda}\right) - G_T^{dB} - G_R^{dB}$)

Exercise 3.2: Two-way wave propagation (entirely reflected wave)

The height of a receiver antenna is 10 *m*. It is used in a radio link where the distance of the transmitter and receiver antenna is 10 *km*. Either increasing or decreasing the height of the receiver antenna, a smaller signal power can be measured on the receiver side. We know that the height of the transmitter antenna is 20 *m*, and the gain of the two antennas is also known, 10 *dB* each.

a) Let us calculate wave length of the radio wave.
b) Let us calculate the attenuation of the radio link.

(Help: $E_R = E_0 + \Gamma_g \cdot E_0 e^{-j2\pi f\Delta/c} = E_0\left(1 + \Gamma_g \cdot e^{-j2\pi\Delta/\lambda}\right)$
where the route difference between the reflected and direct wave is: $\Delta = 2 \cdot h_T h_R / r$
if the reflection is full, then $\Gamma_g = -1$, and so the received electric field strength:
$|E_R| = |E_0| \cdot \left|e^{-j\pi\Delta/\lambda}\right| \cdot \left|e^{+j\pi\Delta/\lambda} - e^{-j\pi\Delta/\lambda}\right| = 2|E_0| \cdot |\sin(\pi\Delta/\lambda)|$, after substitution:
$|E_R| = 2|E_0| \cdot \left|\sin\left(\pi \frac{2h_T h_R}{r \cdot \lambda}\right)\right|$ - <u>*this is the key formula*</u>.
Specifically *in this Exercise*, $\left|\sin\left(2\pi \frac{h_T h_R}{\lambda r}\right)\right| = 1$, so for any integer *k:* $2\pi \frac{h_T h_R}{\lambda r} = \frac{\pi}{2} + k\pi$
Let us investigate only the case when *k=0. Then the wavelength is* 0.08 *m*.)

Exercise 3.3: Two-way wave propagation (partially reflected wave)

A radio transmitter station is given with an operating frequency of 450 *MHz*. In a distance of 3 *km*, we experience that the power of the received signal is in the range of 10 and 90 *nW* - depending on the height of the receiver antenna. If the change in the receiver antenna height is 5m, we measure the same signal power. (The gain of the receiver antenna can be considered as 3 *dB*.)

a) Let us give the explanation of the phenomenon.

b) Let us estimate the height of the transmitter antenna as well as value of the ground reflection coefficient.

(Help: $\left(1+\left|\Gamma_g\right|\right)^2 \Big/ \left(1-\left|\Gamma_g\right|\right)^2 = 90/10$ =9. The valid result is: $\Gamma_g = -0.5$)

Exercise 3.4: Satellite radio links

Let us estimate the gain of a receiver satellite antenna if its physical diameter is 1.1 *m* and the used radio frequency is 12 GHz.

a) How precisely should the antenna be oriented towards a satellite which is moving (staying) on a geostationary orbit?
b) Let us calculate the gain of the satellite antenna if the broadcast program is radiated to a population living in a roughly 2000 *km* diameter circle on the surface of the Earth.

(Help: for very short wavelengths and ellipsoid antennas, the effective area of the antenna can be approximated with the physical area of the antenna circle.)

4^{th} session - Analog modulations

Exercise 4.1: AM-DSB spectral components, signal shapes
Let us give the formula and draw the signal shape of an AM double side band modulated signal if the modulation depth is 50% and 150% and the modulating signal is a sine wave. Draw the demodulated signal for both cases if the demodulation is performed by an envelope detector. Let us calculate the power ratio of side bands and the whole signal. What will be the result if the modulating sine wave is replaced by a symmetric triangle signal (Power ratio, signal shape, spectral description).

Exercise 4.2: AM signal analysis

The output of an AM modulator is the following:

$$s_{AM}(t) = 3\cos(1800\pi t) + 10\cos(2000\pi t) + 3\cos(2200\pi t).$$

Let us determine:

a) The type of AM modulation,
b) $s_m(t)$, the modulating signal,
c) f_c the carrier frequency,
d) The minimal and maximal value of $s_{AM}(t)$,
e) The modulation depth
f) The power ratio of the sidebands and the whole signal.

Exercise 4.3: a typical test exercise

An AM-DSB modulator is given withe following characteristics: 60 kHz carrier frequency, 20% modulation depth, 1.8 V peak value. The modulating signal is a 10 kHz sine wave.
a) Let us draw at least $200\ \mu s$ long section of the modulated signal.
b) Let us give the time function of the modulated signal. What unit should be assigned to the time value?
c) Let us calculate the frequencies and amplitudes of each spectral component.
d) How could this signal be demodulated. Let us draw a block diagram and specify its elements.

Exercise 4.4: unknown modulation

Given an input signal:

$$s_m(t) = 3^{[V]} \cdot \cos(3\pi \cdot t^{[ms]} + 2)$$

a modulator produces the following modulated signal:

$$s_{??}(t) = 4^{[V]} \cdot \cos(500\pi \cdot t^{[ms]} + 21 + 5\sin(3\pi t^{[ms]} + 2))$$

a) What kind of modulation method is used?
b) Let us calculate the amplitude of both the modulating and the modulated signal.

c) Let us calculate the phase and frequency deviation of the modulated signal.
d) Let us estimate the bandwidth of the modulated signal.

5th session - Digital baseband communications

Exercise 5.1: NRZ (Non Return to Zero) signal ideal vs. distorted

a) Let us draw the elementary signal of the ideal, binary NRZ signal. Let us draw the NRZ signal corresponding to the ..,1,0,1,1,.. bit sequence. Let us draw the eye diagram as well.

b) Do the same as in the previous task but this time take dispersion into account as follows: a simplistic model of dispersed NRZ elementary signal where the time function is linearly growing from t=0 to t=T/2. Then the peak value is kept till T, and finally the signal value linearly decreases to zero in t=3T/2. T is the time period.

Exercise 5.2: ISI (Inter Symbol Interference) avoidance

A) Let us calculate the signaling rates (elementary symbols per secundum) appropriate for ISI free communication if the spectrum of the elementary signal is the following:

$$H(f) = \begin{cases} \frac{1}{B} \cdot \left(1 - \left| f/B \right|\right), & \text{if } \left| f \right| \leq B \\ 0, & else \end{cases}$$

B) What is the consequence of a low signaling rate as compared to the bandwidth (B) of the elementary signal.

Exercise 5.3: Matched receiver and transmitter filters

Let us give the optimal receiver and transmitter filters of the previous PAM (Pulse Amplitude Modulation) system. We want reproduce the previous elementary signal (at the receiver side) and we know that the channel is distortionless ($H_C(f) = 1$) and additive white noise is assumable.

Exercise 5.4: Bit/symbol error probability of binary and multilevel PAM systems

An ISI-free binary, synchronous PAM system is given. The values of the samples taken after the receiver filter are ±1.2 V. Wideband Gaussian noise is added to the signal which results in 0 expectation value, 0.3 V variance normal distribution noise added to the samples.

A) Let us calculate the probability of wrong decision (error) due to the additive noise.
B) Let us calculate the error probability of a 4 or 8 level PAM system. It can be assumed that the probability of each symbol is the same.
C) Let us calculate the average power of a multi-level PAM system.

6th session - Digital modulations

Exercise 6.1: OOK vs. BPSK

Compare these two - essentially amplitude modulation - techniques.
A) Draw their time functions assuming an NRZ type of modulating signal.
B) Which one needs more power if we want to achieve the same SNR in both systems. The level of additive noise is the same in both cases and we want equivalent error rates, as well.

Exercise 6.2: The sensitivity of (digital) QAM to amplitude and phase error

Let us investigate the maximum tolerable amplitude and phase error of a 16QAM system.

Exercise 6.3: Power efficient (digital) QAM

Let us investigate the 16 and 64 QAM systems aiming at the reduction of signal power - where it is possible. Note, that the QAM raster must be used. Is it possible for symbols requiring the highest power to be moved to a better (more power efficient) position in the QAM trellis?

7th session - Coding

Exercise 7.1: Source coding
A digital source is given by 10 different symbols. In the most compact binary code of this source, the codeword lengths are the followings: 2,3,3,3,3,4,4,4,5,5.
a) Can this code be a uniquely decodable one?
b) Let us calculate the entropy of the source if we know that - in this specific case(!) - it is equal with average codeword length.

Exercise 7.2: The construction of Shannon code
The source symbols are sorted in a descendant order according to their probability: $p_1 \geq p_2 \geq \dots \geq p_n$. After Shannon, the binary codeword of the *i*-th symbol can be obtained by the truncation of the binary $F_i \in [0,1)$, where the following upper rounding formula keeps the first $l_i = \lceil -\mathrm{ld}\, p_i \rceil$ bits after the comma of the floating point, and

$$F_i = \sum_{k=1}^{i-1} p_k \, .$$

Let us fill the table below.

symb./i	x1/1	x2/2	x3/3	x4/4	x5/5	x6/6
p_i	0.4	0.25	0.12	0.1	0.08	0.05
l_i	2	2	4	4	4	5
F_i	0.0	0.4	0.65	0.77	0.87	0.95
binary F_i	0.00000	0.01100	0.10100	0.11000	0.11011	0.11110
codeword	**00**	**01**	**1010**	**1100**	**1101**	**11110**

We can conclude that Shannon code is prefix but not always optimal as the least significant bit of codeword 6 can be omitted.

Exercise 7.3: Error detection and correction through block coding

The binary, linear, systematic (23,12) Golay code is able to detect 6 errors.

a) Let us calculate the minimum code distance.
b) How many errors can be corrected using this code?
c) How many code words are in this code?
d) How many syndrome vectors are possible when using this code? How many syndromes indicate simple, double and triple errors in a given message?
e) Is it possible to detect quadruple (4 bits) error? Can we make differentiation between triple and quadruple errors?

8th session - Sound and hearing, image and vision

Exercise 8.1: Understanding Fletcher curves

The output of a sine wave function generator drives a loudspeaker. First the frequency of the sound is adjusted to 10 *kHz*. In a given distance an SPL (Sound Pressure Level) meter measures 10*dB*. Then the frequency is decreased to 1 *kHz* and we measure 30 *dB* SPL. Finally the frequency is decreased further to 100*Hz* and the measured SPL is 0 *dB*. Note that the amplitude of the generated sine wave was constant during the experiment.

a) What is the loudness level of the first, 10 *kHz* sound in *phon?*
b) What is the loudness level of the first, 1 *kHz* sound in *phon?*
c) What is the loudness level of the first, 100 *Hz* sound in *phon?*
d) Let us give an explanation of the phenomenon. (Why are the SPL of the three sounds different whilst the generated amplitudes are the same.)

Exercise 8.2: The volume control problem

If we are listening to a music and want to decrease the volume (loudness) by simply lowering the amplitude of the signal we may feel that our sound experience fading. What can be the underlying hearing mechanism?

(Answer: The Fletcher curves are not parallel with each other - especially at low frequencies. This may result in the complete loss of lower music frequencies.)

Exercise 8.3: Colors, luminances

Suzanne wants to make light with her smart phone. She adjust the color of each pixel to green identical to the CIE standard and so the other color LEDs are switched off (the blue and red LEDs of each pixel are operating also with CIE standard wavelengths.)

A) Is this a good idea if only the luminosity counts?
B) How longer can the battery last as compared to the case when all LEDs are "on" producing white light?
C) What is the effectivity of producing white and green lights in terms of luminosity - power ratio?

(Help: To the luminosity, different colors contribute with different weights $Y = 0.3R + 0.59G + 0.11B$. Furthermore, if R=G=B, we are on a gray-scale.)

9th session - Radio Broadcast

Exercise 9.1: Super heterodyne receiver and the mirror frequency

We plan to build a super heterodyne radio receiver. The frequency range of the device should be 87.5 - 108 *MHz* accordingly to the CCIR standard. The intermediate frequency: $F_{IF} = 10.7\ MHz$.

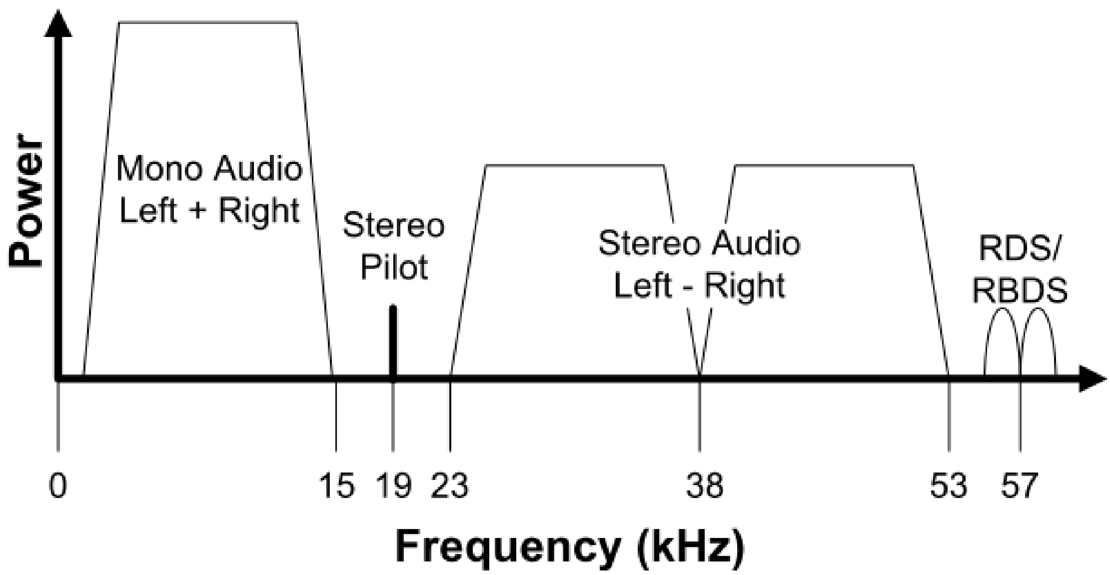
a) Applying *upper* mixing, what should be the frequency range of our local oscillator?
b) How should we tune our local oscillator if we want to receive a radio channel at 94.8 *MHz (*This is "MR2 Petőfi" channel.)
c) What is the image ("mirror") frequency of the previous channel?
d) Why the given intermediate frequency is used for this frequency band?

(Help: upper mixing:,) $f_{oscillator} = f_{received} + F_{IF}$
$f_{mirror} = f_{oscillator} + F_{IF}$

Exercise 9.2: FM Stereo Multiplex signal

a) Let us draw the spectrum of the stereo multiplex signal (this is the modulating signal in the stereo FM transmitter).
b) Let us calculate the frequency components of a stereo multiplex signal if the left input signal is a 800 Hz sinewave, and the right stereo channel is completely silent.
c) Using an older, mono FM receiver, can we understand the speech in a stereo FM broadcast signal?

(Help:



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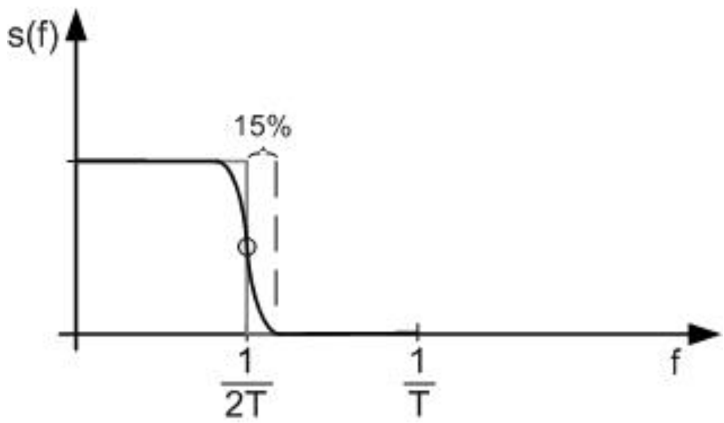
10^{th} session - Digital Video Broadcast

Exercise 10.1: DVB-C

We have a bandwidth of 8MHz in a cable-TV system to transfer digital data instead of one analogous TV channel. How many digital TV channels can be transmitted in the given bandwidth, applying the following conditions: 64-QAM, the spectrum of the elementary signal used for digital modulation is 15% raised cosine one. For error detection, the RS(204,188) Reed-Solomon coding is used.

(Help:



so, the bandwidth of the QAM: $2 \cdot \frac{1}{2T} 1.15 = 8MHz$, where the symbol rate is 1/T.

From this, symbol rate: $\frac{1}{T} = 6.96 MBaud$

So, the bit rate is *6* bits/symbol x *6,96* million symbol/sec=*47.41* Mbps

Net bit rate: *41.74* Mbps · *188/204= 38.47*Mbps

Assuming 1.5Mbps/digital TV channel,
the total number of channels is $\lfloor 38{,}74/1.5 \rfloor = 25$.)

Exercise 10.1: DVB-T

Let us calculate the net bit rate of a digital terrestrial video broadcast system if the following parameters are given:

- OFDM with 6817 carriers, from which 6048 carry real data. (The remaining ones are pilot signals.)
- The symbol time, T_0 is 896 μs.
- The guard interval for each symbol time: GI=+1/4.
- 64QAM around each real data carrier.
- Coding algorithm used for error detection: RS(204,188)
- The ratio of “Forward Error Correction” used to add further redundancy in order to make the system more robust against noise: FEC-rate=3/4

(Help:

Total symbol rate: $6048 \cdot \frac{1}{(1+1/4)T_0} = 5.4 MBaud$ - where ¼ stands for GI (Guard Interval), and T_0=896 μs.

Total bit rate: 5.4MBaud · 6 bits/symbol = 32.4Mbps.

Net bit rate (normalizing with FEC=3/4, RC(204/188)): 32.4Mbps · 3/4 · 188/204 = 22.4Mbps.

This is enough for 14-15 digital TV channels)

11th session - GSM

Exercise 11.1: Cells, clusters

Let us estimate the potential number of active cellular phone users in Budapest. The full 900 MHz GSM band is available, but it is partitioned between 3 service providers. Let us calculate the maximum number of active users in a given cell, at a given service provider.

Conditions: the area of Budapest is 525 km^2. Frequency Reuse Factor: N=12. Total number of cells: 525. Average area of a cell: 1km^2. ~40 FDM channels/service provider. Bands: 890-915MHz for uplink, 935-960MHz for downlink.

(Help:
Cluster size is 12 cells.
Num of clusters = $\left\lceil \frac{525\text{km}^2}{12\text{x}1\text{km}^2} \right\rceil = 44$

Num of FDM channel per cluster = (915-890)MHz / 200 kHz = 125.
Num of TDM channels per FDM channels is 8)

Exercise 11.2: Network subsystem, call set-up

Let us draw the essential components of a GSM system. Analyze the set-up of a call.

(Help: MS, BTS, BSC, MSC+VLR, HLR, GMSC, AuC, FNR, EIR,...)

12h session - VoIp, Traffic scaling

Basic definition for traffic scaling:

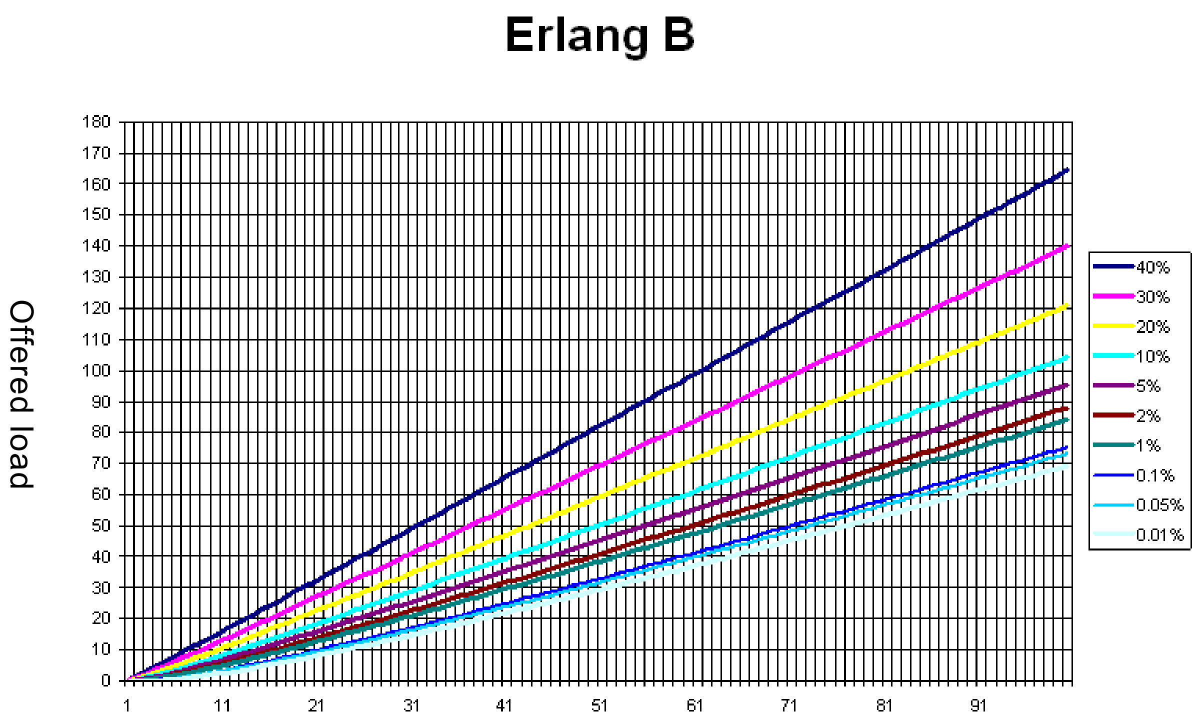
- Offered load: $A = h \cdot \lambda$,

where,

- h: average call holding time
- λ: average call arrival rate

If the number of servers (interface circuits, operators, etc.) is N,

then the probability of blocking, P_b can be calculated by using the Erlang B formula. (Or using the figure below.)



Num of servers

- The average occupancy of one server: a = A/N x [1-P_B].

- The cumulative distribution function of the exponential probability distribution can be used to calculate the probability that a given call length is not longer than x.

$$F(x) = \begin{cases} 1 - e^{-\lambda x} & , \ x \geq 0, \\ 0 & , \ x < 0. \end{cases}$$

Please note, that here the parameter of the distribution is not the lambda of the call arrival rate, but the reciprocant of the avg holding time.